Veszprém Conference on Differential and Difference Equations and Applications

Abstracts

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Dichotomy and optimal control

Pavel Brunovský
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We discuss variational necessary conditions of optimality (such as e.g. Pontjagin maximum principle) for infinite horizon discrete time optimal control problems. Such conditions can bee obtained by the Langrange formalism, provided a certain operator has closed range. This condition is satisfied if the linearized dynamics along the tested trajectory has exponential dichotomy, i.e., it splits into an exponentially expanding and an exponentially contracting components.

The dichotomy hypothesis limits the applicability of the result to particular examples severely. We show that it can be relaxed: the dynamics can have a component which is neither contracting nor expanding, provided this component is fully controllable.

Stable periodic orbits for the Mackey–Glass equation

Tibor Krisztin
University of Szeged, Hungary

We study the classical Mackey–Glass delay differential equation

$$x'(t) = -ax(t) + bf_n(x(t-1))$$

where a, b, n are positive reals, and $f_n(u) = \frac{u}{1+u^n}$ for $u > 0$. As a limiting ($n \to \infty$) case we also consider the discontinuous equation

$$x'(t) = -ax(t) + bf(x(t-1))$$

where $f(u) = u$ for $u \in (0,1)$, $f(1) = 1/2$, and $f(u) = 0$ for $u > 1$. First, for certain parameter values a, b, an orbitally asymptotically stable periodic orbit is constructed for the discontinuous equation. Then it is shown that for large values of n, and with the same parameters a, b, the Mackey–Glass equation also has an orbitally asymptotically stable periodic orbit near to the periodic orbit of the discontinuous equation.

In spite of the orbital stability, the projection of the periodic solutions into the plane $t \mapsto (x(t), (x(t-1)))$, can be complex.

Joint work with G. Kiss and A. Vigh (Szeged).

Nonautonomous bifurcations

Christian Pötzsche
Alpen-Adria Universität Klagenfurt, Austria

Various real-world problems lead to *nonautonomous* differential or difference equations. That is, they involve time-dependent parameters, controls, modulation and various other effects. Special cases include periodically or almost periodically forced systems, but in principle the time dependence can be arbitrary. As a consequence, many of the now well-established concepts, methods and results from the classical theory for autonomous dynamical systems are no longer applicable and require appropriate extensions.

We discuss several approaches to obtain a bifurcation theory for explicitly time-variant equations. They include an appropriate spectral theory or nonautonomous counterparts to equilibria and attractors.

Differentiability in Fréchet spaces and delay differential equations

Hans-Otto Walther
Universität Gießen, Germany

In infinite-dimensional spaces there are non-equivalent notions of continuous differentiability which can be used to derive the familiar results of calculus up to the Implicit Function Theorem and beyond. For autonomous differential equations with variable delay, not necessarily bounded, the search for a state space in which solutions are unique and differentiable with respect to initial data leads to smoothness hypotheses on the vector functional f in an equation of the general form

$$x'(t) = f(x_t), \quad x_t(s) = x(t+s) \quad (s \leq 0),$$

which have implications (a) on the nature of the delay (which is hidden in f) and (b) on the type of continuous differentiability which is present. The lecture discusses these issues, introduces the appropriate *strong* kind of continuous differentiability, and presents the associated results on differentiable solution operators on a Fréchet manifold, with local invariant manifolds at equilibria.

On nonlinear surface growth models

Gabriella Bognár
University of Miskolc, Hungary

The technique of growth surfaces under Molecular Beam Epitaxy has received considerable attention for a wide range of technological and industrial applications. This approach provides unique capability to grow crystalline thin films with precise control of thickness, composition and morphology. This enables scientists to build nanostructures as pyramidal objects or mounds. The evolution of the surface morphology during MBE growth results from a competition between the molecular flux and the relaxation of the surface profile through surface diffusion. The aim of this talk is to describe the coarsening process, to present results showing that surfaces can be mathematically and physically classified into different categories, to provide analytical results on the wavelength and amplitude. Numerical simulations are presented to show the roughening and coarsening of the surface pattern and the evolution of the surface morphology in time for different parameter values in one- and in two-dimensions.

Stochastic 2D Navier-Stokes equations with infinite delay

Tomás Caraballo
Universidad de Sevilla, Spain

In this hereditary characteristics. The existence and uniqueness of solutions in the case of unbounded (infinite) delay are first proved by using the classical technique of Galerkin ap The local stability analysis of constant solutions (equilibria) is also carried out by exploiting several approaches. Namely, the Lyapunov function method, the Razumikhin-Lyapunov technique and by constructing appropriate Lyapunov functionals. Although, in general, it is not possible to establish conditions ensuring the exponential asymptotic behavior of the solutions, some sufficient conditions for the polynomial stability of the stationary solution in a particular case of unbounded variable delay will be provided. Many other interesting cases of unbounded delay terms remain as open problems. Also the global asymptotic behavior is an interesting topic which is being investigated.

Qualitative properties of linear autonomous delay differential systems: Integer versus non-integer order case

Jan Čermák
Institute of Mathematics, Brno University of Technology, Czech Republic

We discuss basic qualitative properties of a linear fractional differential system

$$\mathrm{D}^{\alpha} y(t) = Ay(t-\tau), \qquad t > 0 \tag{1}$$

where D^{α} means the Caputo derivative of a positive real order α, A is a constant real matrix and τ is a positive real constant delay. While basic stability, asymptotic and oscillation results for (1) with a positive integer α are mostly well-known, their fractional analogues are still the subject of a current research. We give a survey of recent results on this topic, with a special emphasize put on similarities and dissimilarities between integer and fractional order case. Also, we mention some comments on corresponding fractional difference systems.

Large-time behaviour of solutions of delayed-type linear differential equations

Josef Diblík
Brno University of Technology, Czech Republic

Asymptotic behaviour of solutions of delayed-type linear differential functional equations with bounded delays $\dot{x}(t) = -L(t, x_t)$ is analyzed when $t \to \infty$. The main results concern the existence of two significant positive and asymptotically different solutions $x = \varphi^*(t)$, $x = \varphi^{**}(t)$ such that $\lim_{t\to\infty} \varphi^{**}(t)/\varphi^*(t) = 0$. These solutions make it possible to describe the family of all solutions by means of an asymptotic formula. The investigation basis is formed by an auxiliary linear differential functional equation of retarded type $\dot{y}(t) = L^*(t, y_t)$ such that $L^*(t, y_t) \equiv 0$ for an arbitrary constant initial function y_t. Results are applied to investigation large-time behaviour of positive solutions to a generalized Dickman equation. A criterion is also given for sufficient conditions on initial functions to generate positive solutions with prescribed asymptotic behaviour with values of their weighted limits computed.

Positivity for non-Metzler systems

Alexander Domoshnitsky
Ariel University, Israel

In 1950, Wazewski obtained the following necessary and sufficient condition for positivity of linear systems of ordinary differential equations: the matrix of coefficients is Metzler (i.e. non-diagonal elements in the matrix of coefficients are nonnegative). No results on the positivity of solutions to delay systems in the case where the matrix is non-Metzler were expected to be obtained. It was demonstrated that for delay systems the Wazewski condition does not work as a necessary one. We prove results on positivity of solutions for non-Metzler systems of delay differential equations. New explicit tests of the exponential stability are obtained as applications of results on positivity for non-Metzler systems. Examples demonstrate possible applications of our theorems to stabilization. For instance, in view of our results, the implicit requirement on dominance of the main diagonal can be skipped. Our approach is based on nonoscillation of solutions and positivity of the Cauchy functions of scalar diagonal delay differential equations.

Asymptotic problems for second order functional differential equations

Zuzana Došlá
Masaryk University, Brno, Czech Republic

We study the existence of positive decreasing solutions (so called Kneser solutions) for the functional differential equation with damping term

$$x''(t) = h(t, x(\gamma(t), x(t)))x'(t) + f(t, x(\tau(t)), x(t)).$$

The existence of zero-decaying Kneser solutions is proved, too.

We illustrate the role of the deviating argument to the asymptotic behavior of solutions for

$$x''(t) = b(t)|x(\tau(t))|^{\beta} \operatorname{sgn} x(\tau(t)), \quad \tau(t) < t,$$

in both cases $\beta > 1$ and $\beta < 1$.

This is a joint work with Petr Liška, Mendel University in Brno, and Mauro Marini, University of Florence.

Reliable numerical models in epidemic propagation

István Faragó and Róbert Horváth
ELTE and BUTE, Hungary

Miklós E. Mincsovics
BUTE, Hungary

In order to have an adequate model, the continuous and the corresponding numerical models on some fixed mesh should preserve the basic qualitative properties of the original phenomenon. In the talk we focus our attention on some discrete mathematical models of biology, namely we consider some continuous and discrete epidemic models and we will investigate their qualitative properties. First we investigate the SIR model. We give those conditions for the continuous and finite difference discrete models, under which the nonnegativity and some other basic qualitative properties (mass conservation, monotonicity) are valid. Special attentions will be paid to the propagation of malaria. We formulate and investigate different discrete models and we give sufficient conditions for the non-negativity property. Numerical examples demonstrate the sharpness of the results.

Asymptotic behaviour for some classes of non-monotone delay differential equations

Teresa Faria
Universidade de Lisboa, Lisboa, Portugal

We study the global asymptotic behaviour of solutions for some families of n-dimensional non-autonomous delay differential equations (DDEs), which encompass a large number of structured population models. Sufficient conditions for both the extinction of all the populations and the permanence of such systems are given [2]. The case of periodic systems is further analysed [1].

1. T. Faria, Periodic solutions for a non-monotone family of delayed differential equations with applications to Nicholson systems, *J. Differential Equations* 263 (2017), 509–533.
2. T. Faria, R. Obaya, A.M. Sanz, Asymptotic behaviour for a class of non-monotone delay differential systems with applications, *J. Dyn. Diff. Equ.*, (on-line). DOI 10.1007/s10884-017-9572-8

New paths of path integrals

Márcia Federson
Universidade de São Paulo, Brazil

We retake some intricacies behind the Feynman path integral and the physics involved. Then we resume the main ideas of the non-absolute path integral defined by Ralph Henstock, which fixes the main deficiency of the Feynman integral, and consider applications to electromagnetism.

Delay difference equations: permanence and the structure of the global attractor

Ábel Garab
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In the first part of the talk we give sufficient conditions on the uniform boundedness and permanence of non-autonomous multiple delay difference equations of the form

$$x_{k+1} = x_k f_k(x_{k-d}, \dots, x_{k-1}, x_k),$$

where $f_k \colon D \subseteq (0,\infty)^{d+1} \to (0,\infty)$. This also implies the existence of the global (pullback) attractor, provided the right-hand side is continuous.

In the second part, under some feedback conditions on g, we give a so-called Morse decomposition of the global attractor for equations of the form $x_{k+1} = g(x_{k-d}, x_k)$. The decomposition is based on an integer valued Lyapunov functional introduced by J. Mallet-Paret and G. Sell.

Both of our results are applicable for a wide range of single species discrete time population dynamical models, such as models by Ricker, Pielou, Mackey–Glass, Wazewska–Lasota, and Clark.

Partially joint work with Christian Pötzsche (AAU Klagenfurt).

Oscillation results for even-order nonlinear differential equations with a sublinear neutral term

John R. Graef
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The authors present a new technique for the linearization of even-order nonlinear differential equations with a sublinear neutral term. They establish some new oscillation criteria via comparison with higher-order linear delay differential inequalities as well as with first-order linear delay differential equations whose oscillatory characters are known. Examples are provided to illustrate the theorems.

Limiting behaviour of an SIS epidemic model with environmental stochasticity

David Greenhalgh, Jiafeng Pan, Alison Gray and Xuerong Mao
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We extend the classical SIS (susceptible-infected-susceptible) epidemic model from a deterministic to a stochastic one and formulate it as a stochastic differential equation (SDE) for $I(t)$, the number of infectious individuals at time t. The stochasticity is introduced as a Brownian motion in the disease transmission coefficient (equivalently in the contact rate of infected individuals). This models the effect of random environmental variation. After deriving the SDE for the spread of the disease we then prove that this SDE has a unique positive solution. For the deterministic model classical results show that there is a unique threshold value R_0^D, the deterministic basic reproduction number, such that if R_0^D is less than or equal to one then the disease will die out and otherwise tends to a unique endemic equilibrium. We show that for the stochastic model there is a smaller threshold value R_0^S and provided that a condition involving the variance of the stochastic noise is satisfied then the disease will die out almost surely (a.s.) for $R_0^S < 1$. If $R_0^S > 1$ then we show that the disease will fluctuate about a strictly positive level a.s. We then show that if $R_0^S > 1$ the SDE SIS model has a unique non-zero stationary distribution and derive expressions for the mean and variance of this stationary distribution. All the theoretical results are illustrated and confirmed by numerical simulations. We finish by discussing two real-life examples.

Approximation and delayed chemical reaction networks

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The mathematical modelling of spatially distributed dynamic phenomena, such as convection and diffusion, leads to partial differential equation models, that are commonly approximated in engineering practice using discretization schemes in the spatial variable(s). This may lead to various forms of delayed differential equations when chemical reactions take place in combination with the above phenomena.

Based on the underlying physical and chemical laws (e.g. conservation) and on the properties of the approximating numerical schemes, the origin of delayed differential equation models with different (e.g. discrete, distributed) delays will be shown, and the properties of the resulting delayed chemical reaction network models will be analysed.

Furthermore, results on the approximation error and on the stability properties of delayed chemical reaction network models with special properties will also be presented.

Models for human balancing with reaction delays

Tamás Insperger
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Maintaining balance is a vital ability for humans: falls are leading causes of accidental death and morbidity in the elderly. Studies on human balancing help to understand the feedback control mechanisms employed by the human nervous system, which may lead to conclusions that decrease the amount of future accidents related to balance problems. Different mechanical models of human balancing are investigated: (1) stick balancing on the fingertip; (2) stick balancing on a linearly driven cart; (3) balancing on a balance board in the AP direction; (4) balancing on a balance board in the ML direction; and (5) balancing an object rolling on a see-saw (ball-and-beam system). A key element of the models of these balancing tasks is the reaction time delay. The control algorithm employed by the human nervous system for balancing tasks in the presence of reaction delay has not been fully explored yet. Several control concepts arise in the literature as possible candidates: predictor feedback, event- or time-driven intermittent controllers, and acceleration feedback can be mentioned as examples. Here, one of the simplest stabilizing controllers, a delayed proportional-derivative feedback controller is considered. Stability and stabilizability conditions of the different models are analyzed numerically and are illustrated using stability diagrams. The results of the numerical investigations of the models are compared to real balancing trials performed by human subjects.

Finding periodic solutions without finding eigenvalues

Benjamin B. Kennedy
Gettysburg College, USA

We describe the structure of the set of periodic solutions of the nth-order linear scalar-valued difference equation

$$y(k) = a_1 y(k-1) + a_2 y(k-2) + \cdots + a_n y(k-n)$$

in the case that the coefficients a_i are all rational. This description yields an elementary method for finding periodic solutions that does not involve computing zeros of the characteristic polynomial.

On exponential stability of fractionally perturbed ODEs

Milan Medveď
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We discuss stability properties of fractional differential equations and the following fractionally perturbed ODEs:

$$\dot{x}(t) = Ax(t) + f\Big(t, x(t), I^{\alpha_1}x(t), \ldots, I^{\alpha_m}x(t)\Big),\ t > 0, x(t) \in R^N, \qquad (1)$$

where A is a constant matrix and $I^{\alpha_i}x(t) := \frac{1}{\Gamma(\alpha_i)}\int_0^t (t-s)^{\alpha_i - 1}x(s)ds$, $0 < \alpha_i < 1$, $i = 1, 2, \ldots, m$ are the Riemann-Liouville fractional integrals of a function $x(t)$ of orders α_i, $f : R \times R^N \times R^{mN} \to R^N$. It is well known that the linear fractional differential equation $D^\alpha x(t) = Ax(t)$, $0 < \alpha < 1$, do not have exponenially stable equilibrium $x = 0$. It is asymptotically stable if and only if $|\arg(spec\,(A))| > \frac{\alpha\pi}{2}$. In this case all components of $x(t)$ decay towards 0 like $t^{-\alpha}$ as $t \to \infty$. We show that the fractional perturbations of ODEs of the type (1) can have exponentially stable solutions and we present some results on the exponential stability of the zero solution of the equation (1).

The talk is based on a joint work with Eva Brestovanská, Faculty of Managament, Comenius University, Bratislava.

Asymptotic properties of solutions to difference equations of Volterra type

Janusz Migda
Faculty of Mathematics and Computer Science, A. Mickiewicz University, Poland

We present a new approach to the theory of asymptotic properties of solutions to discrete Volterra equations of the form

$$\Delta^m x_n = b_n + \sum_{k=1}^{n} K(n,k) f(k, x_k), \quad f : \mathbb{N} \times \mathbb{R} \to \mathbb{R},$$

and more general equations of the form

$$\Delta^m x_n = b_n + \sum_{k=1}^{n} K(n,k) F(x)(k), \quad F : \mathbb{R}^{\mathbb{N}} \to \mathbb{R}^{\mathbb{N}}.$$

Our method is based on using the iterated remainder operator and asymptotic difference pairs. This approach allows us to control the degree of approximation.

Properties of solutions to second-order neutral difference equations

Małgorzata Migda
Institute of Mathematics, Poznań University of Technology, Poland

We study asymptotic behavior of solutions to second-order neutral difference equation of the form

$$\Delta(r_n \Delta(x_n + p_n x_{n-\tau})) = a_n f(n, x_n) + b_n,$$

where τ is a nonnegative integer, (r_n) is a sequence of positive real numbers, (a_n), (b_n) and (p_n) are sequences of real numbers, and f is a real function. We present sufficient conditions, under which for an arbitrary real constant c there exists a solution of the above equation convergent to c.
Moreover, using the discrete Bihari type lemma and discrete L'Hospital's type lemma, we obtain sufficient conditions, under which all nonoscillatory solutions have the property $x_n = cr_n^* + o(r_n^*)$, or the property $x_n = cr_n^* + d + o(1)$ where $r_n^* = \sum_{i=1}^{n-1} \frac{1}{r_n}$. These results are new even for linear equations of the considered form, and in the case when $p_n \neq 0$.

Exponential separation for linear differential systems

Flaviano Battelli
Marche Polytechnic University, Italy

Kenneth Palmer
National Taiwan University, Taiwan

An important concept in the study of nonautonomous linear differential systems is that of exponential separation. It is closely related to the concept of exponential dichotomy. Also it has played a key role in the theory of Lyapunov exponents. Usually in the study of exponential separation, it is assumed that the coefficient matrix is bounded in norm. Our first aim here is to develop a theory of exponential separation which applies to unbounded systems. It turns that in order to have a reasonable theory, it is necessary to add the assumption that the angle between the two separated subspaces is bounded below. This is what is meant by strong exponential separation. Our second aim is to show that if a bounded linear Hamiltonian system is exponentially separated into two subspaces of the same dimension, then it must have an exponential dichotomy. We also describe similar results for linear difference equations.

A nonstandard "discretization" technique for the solution of ODEs

Eugenia N. Petropoulou
University of Patras, Greece

A nonstandard "discretization" technique for the solution of ordinary differential equations (ODEs) will be presented. This technique is based on the transformation of the original ODE into an equivalent difference equation, through an operator equation of an abstract Banach space. The technique will be demonstrated by applying it to initial or boundary value problems arising in oscillation problems, population problems, as well as in problems in fluid mechanics. The obtained computed solutions are both real and complex.

Approximating Volterra equations using equations with piecewise constant arguments

David Reynolds
Dublin City University, Ireland

In 1984 and 1986, Cook and Wiener published papers on equations with piecewise continuous arguments. In 1991, Győri showed how some solutions to delay differential equations equations can be approximated using functions with piecewise constant argument. Interest in this topic has grown considerably. This presentation explains how these ideas can applied to Volterra integral equations, and in particular to scalar linear equations of the second kind.

This is joint work with Győri István of the University of Pannonia at Veszprém.

Dynamics of new delay logistic equations arising from cell biology

Gergely Röst
University of Oxford & University of Szeged

Joint work with Ruth Baker (Oxford) and Péter Boldog (Szeged).

The delay logistic equation, originating from Hutchinson, has played a crucial role in the theory of nonlinear delay differential equations, inspiring the development of a variety of methods. However, the equation has received criticisms from biological modellers due to the lack of a mechanistic derivation. In this talk we present a new delay logistic equation with clear biological underpinning from cell population dynamics, motivated by the go or grow type behaviour of glioma cells. First we construct some individual based stochastic models under different biological assumptions, where the cell cycle length is explicitly incorporated, and investigate their behaviours.

Then the mean field equations are derived, and they turn out to be logistic type delay differential equations with discrete and distributed delays. For the most challenging case, we provide a complete global analysis of the equation showing global asymptotic stability of the positive equilibrium, based on persistence argument, comparison principle and an L2-perturbation technique. Yet, the dynamics is not trivial, as there exist very long transients with oscillatory patterns of various shapes, for which we offer some explanations.

We also show that if we add an instantaneous positive feedback term to the classical delay logistic equation, then local stability does not imply global stability so a Wright-type conjecture is not valid any more (this last part is from a joint work with István Győri, Veszprém and Yukihiko Nakata, Shimane).

Leader-following problem on times scales

Ewa Schmeidel
University of Bialystok, Poland

We propose and prove conditions ensuring a consensus in the leader-following problem. The main idea of the leader-following problem is to drive a team of agents to reach an agreement on a certain issue by negotiating with their neighbors. In more details, each agent receives information from the set of other agents in the group and then all agents adjust their own information (opinion) states depending on the information received from other agents. The system of equations describing the leader-following problem is considered on time scales, particularly discrete time scales. Numerical examples are provided to illustrate the results.

Some multiplicity results for boundary value problems with singular ϕ-Laplacian

Călin Şerban
West University of Timişoara, Romania

We study the existence of multiple periodic solutions for boundary value problems with singular ϕ-Laplacian operator and Fisher-Kolmogorov type nonlinearities. The approach is variational and relies on a generalization of a result for smooth functionals of Clark to convex, lower semicontinuous perturbations of C^1- functionals due to Szulkin. The talk is based on joint work with Petru Jebelean.

Acknowledgments. This presentation is supported from the grant of Ministery of Research and Innovation, CNCS - UEFISCDI, project number PN-III-P1-1.1-PD-2016-0040, title "*Multiple solutions for systems with singular ϕ-Laplacian operator*", within PNCDI III.

Multiple solutions of nonlinear elliptic functional differential equations

László Simon
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We shall consider weak solutions of boundary value problems for elliptic functional differential equations of the form

$$-\sum_{j=1}^{n} D_j[a_j(x,u,Du;u)] + a_0(x,u,Du;u) = f, \quad x \in \Omega$$

with homogeneous boundary conditions, where $\Omega \subset \mathbb{R}^n$ is a bounded domain and $;u$ denotes nonlocal dependence on u (e.g. integral operators applied to u).

By using the theory of pseudomonotone operators, one can prove existence of solutions.

However, in certain particular cases it is possible to find theorems on the number of solutions. These statements are based on arguments for fixed points of certain real functions and operators, respectively.

The effect of graph properties on the threshold in network models

Péter L. Simon
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Simple compartmental epidemic models exhibit transcritical bifurcation as the ratio of the infection and recovery rates are varied. When the infection rate (τ) is small, then the disease-free steady state is globally stable, while above a critical value it becomes unstable and a globally stable endemic steady state appears. The goal of the talk is to compare this critical value for different models of SIS epidemic spread on networks. Denoting by γ the recovery rate, the threshold of the simplest model is at $n\tau = \gamma$, where n is the average degree of the graph. In the case of the so-called pairwise model it is $(n-1)\tau = \gamma$. For the degree-based mean-field model the threshold is at $\tau\langle d^2\rangle = \gamma\langle d\rangle$, where $\langle d^k\rangle$ is the k-th moment of the degree distribution. For the individual-based mean-field model the threshold is at $\Lambda\tau = \gamma$, where Λ is the leading eigenvalue of the adjacency matrix.

Invariant regions for systems of lattice reaction-diffusion equations

Antonín Slavík
Charles University, Czech Republic

We study systems of lattice differential equations (i.e., equations with discrete space and continous time) of reaction-diffusion type. Such systems frequently appear in population dynamics (e.g., predator-prey models with diffusion).

After establishing some basic properties such as the local existence and global uniqueness of bounded solutions, we proceed to our main goal, which is the study of invariant regions. Our main result can be interpreted as an analogue of the weak maximum principle for systems of lattice differential equations. It is inspired by existing results for parabolic differential equations, but its proof is different and relies on the Euler approximations of solutions to lattice differential equations. As a corollary, we obtain a global existence theorem for nonlinear systems of lattice reaction-diffusion equations.

Approximating the domain of attraction of uncertain nonlinear systems using linear matrix inequalities and Finsler's lemma

Péter Polcz and Gábor Szederkényi
Pázmány Péter Catholic University, Hungary

Tamás Péni
Institute for Computer Science and Control (MTA SZTAKI), Hungary

In this contribution, we consider locally asymptotically stable nonlinear autonomous models in ODE form, where the coordinates functions of the right hand side are rational functions of the differential variables and may contain constant uncertain parameters. It is assumed that the initial values and the uncertain parameters belong to known polytopes. The rational terms contained in the Lyapunov function are computed using the linear fractional representation (LFR), and the required properties of the Lyapunov function are given in the form of linear matrix inequalities. The domain of attraction (DOA) is approximated using the appropriate level sets of the computed Lyapunov function. To add further degrees of freedom to the computations, we use Finsler's lemma, which requires the determination of affine annihilators for the basis functions. To decrease the size of the resulting optimization problem without increasing the level of conservatism of the DOA estimation, we propose simplification algorithms for the LFR and for the annihilator. The operation of the method is shown on illustrative examples known from the literature.

Saddle-node bifurcation of periodic orbits for a delay differential equation

Gabriella Vas
University of Szeged, Hungary

We consider the scalar delay differential equation

$$\dot{x}(t) = -x(t) + f_K(x(t-1))$$

with a nondecreasing feedback function f_K depending on a parameter K. As it is well-known, periodic orbits arise via a series of Hopf-bifurcations for strictly monotone increasing nonlinearities, e.g. for $f_K = K\tanh(x)$ as K increases. Other types of bifurcations involving periodic orbits are rarely studied. Now we verify that for suitably chosen f_K, a saddle-node bifurcation of periodic orbits takes place as K varies.

The nonlinearity f_K is chosen so that it has two unstable fixed points (hence the dynamical system has two unstable equilibria), and these fixed points remain bounded away from each other as K changes. The generated periodic orbits are of large amplitude in the sense that they oscillate about both unstable fixed points of f_K.

This is a joint work with Szandra Guzsvány.

On a first-order partial differential equation

Tamasha Aldibekov
Al-Farabi Kazakh National Univesrsity, The Republic of Kazakhstan

First-order partial differential equation is considered

$$\frac{\partial u}{\partial x} + \sum_{k=1}^{n} p_{1k}(x) y_k \frac{\partial u}{\partial y_1} + \ldots + \sum_{k=1}^{n} p_{nk}(x) y_k \frac{\partial u}{\partial y_n} = 0 \qquad (1)$$

where $u(x, y_1, \ldots, y_n)$ is unknown function, $x_0 \leq x < +\infty)$, $x_0 > 0$, $-\infty < y_1, \ldots, y_n < +\infty$; $p_{ik}(x)$, $i = 1, \ldots, n; k = 1, \ldots, n$ has continuous first order partial derivatives on $x_0 \leq x < +\infty)$. For (1) exists integral basis $\varphi_k(x_0, x, y_1, \ldots, y_n), k = 1, \ldots, n$ and if it attempts to zero on $x_0 \to +\infty$, then perturbed first-order partial differential equation (2)

$$\frac{\partial u}{\partial x} + \left(\sum_{k=1}^{n} p_{1k}(x) y_k + g_1(x, y_1, \ldots, y_n) \right) \frac{\partial u}{\partial y_1} +$$

$$\ldots + \left(\sum_{k=1}^{n} p_{nk}(x) y_k + g_n(x, y_1, \ldots, y_n) \right) \frac{\partial u}{\partial y_n} = 0 \qquad (2)$$

has an integral basis, which attempts to zero on $x_0 \to +\infty$, then a linear homogeneous first order partial differential equation (1) is said to be asymptotically stable on $x_0 \to +\infty$. It is established that if:
1)$p_{ik}(x), i = 1, \ldots, n,\ k = 1, \ldots, n$; are continuous differentiable on I.
2)$p_{k-1,k-1}(x) - p_{kk}(x) \geq \alpha\varphi(x),\ x \in I,\ k = 2, \ldots, n,\ \alpha > 0,\ \varphi(x) \in C(I)$, $\varphi(x) > 0,\ q(x) = \int\limits_{x_0}^{x} \varphi(s) ds \uparrow +\infty$;
3) $\lim\limits_{x \to +\infty} \frac{|p_{ik}(x)|}{\varphi(x)} = 0,\ i \neq k,\ i = 1, 2, \ldots, n,\ k = 1, 2, \ldots, n$;
4) $\lim\limits_{x \to +\infty} \frac{1}{q(x)} \int\limits_{x_0}^{x} p_{kk}(s) ds = \beta_k,\ k = 1, 2, \ldots, n,\ \beta_1 < 0$;
5)$g(x, y) = colon\,(g_1(x, y_1, \ldots, y_n), \ldots, g_n(x, y_1, \ldots, y_n))$ has a continuous partial derivatives on the set $x_0 \leq x < +\infty,\ x_0 > 0,\ -\infty < y_1, \ldots, y_n < +\infty$, $g_i(x, 0, \ldots, 0) = 0,\ i = 1, \ldots, n$. $\|g(x, y)\| \leq \delta(x)\|y\|$, $\lim\limits_{x \to +\infty} \frac{\delta(x)}{\varphi(x)} = 0$, then the equation (1) is asymptotically stable

1. P. Hartman Ordinary differential equations. New York. 1962

Global stability in a system using echo for position control

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We consider a system of equations describing automatic position control by echo. The system can be reduced to a single differential equation with state-dependent delay. The delayed terms come from the control mechanism and the reaction time. H.-O. Walther [1] proved that stable periodic motion is possible for large enough reaction time. We show that, for sufficiently small reaction lag, the control is perfect, i.e., the preferred position of the system is globally asymptotically stable.

1. H–O. Walther, Stable periodic motion of a system with state dependent delay, Differ. Integral Equ. 15(2002), No. 8, 923–944.

Exponential stability of linear discrete systems with multiple delays

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The paper investigates the exponential stability and exponential estimate of the norms of solutions to a linear system of difference equations with multiple delays

$$x(k+1) = Ax(k) + \sum_{i=1}^{s} B_i x(k - m_i), \qquad k = 0, 1, \ldots$$

where $s \in \mathbb{N}$, A and B_i are square matrices and $m_i \in \mathbb{N}$.

New criterion for exponential stability is proved by the Lyapunov method.

An estimate of the norm of solutions is given as well and relations to the well-known results are discussed. Results are illustrated by an example.

Distributed control in stabilization of a model of infection diseases

Alexander Domoshnitsky, Marina Bershadsky and Irina Volinsky
Ariel University, Israel

In this talk we consider a model of infection diseases built by G.I. Marchik in the form of ordinary differential system

$$\begin{cases} \frac{dV}{dt} = \beta V(t) - \gamma F(t)V(t), \\ \frac{dC}{dt} = \zeta(m)\alpha F(t-\tau)V(t-\tau)\Theta(t-\tau-\tau_0) - \mu_c(C(t)-C^*), \\ \frac{dF}{dt} = \rho C(t) - \eta\gamma F(t)V(t) - \mu_f F(t), \\ \frac{dm}{dt} = \sigma V(t) - \mu_m m(t), \end{cases}$$

Here $V(t)$ is antigen concentration rate, $C(t)-$ plasma cell concentration rate, $F(t)-$ antibody concentration rate, C^* and F^* are the plasma rate concentration and antibody concentration of the healthy body respectively, $m(t)$ is relative features of the body.

Parameters α, β, γ, μ_f, μ_m, μ_c, σ, ρ, η are given parameters, $\zeta(m)$, $\Theta(t-\tau-\tau_0)$ are given functions.

We demonstrate that the control

$$u(t) = -b\int_0^t (F(s) - F^* - \varepsilon)e^{-k(t-s)}ds,$$

where b, k, e are positive constants can stabilize this system in the neighborhood of a stationary solution.

Initialization of homoclinic solutions near Bogdanov-Takens points in delay differential equations

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Hasselt University, Belgium

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Great interest has been shown in the continuation of homoclinic bifurcation curves emanating from codimension 2 Bogdanov-Takens bifurcation points in ordinary differential equations. In this talk, the parameter-dependent center manifold reduction near the generic and transcritical codimension 2 Bogdanov-Takens bifurcation in delay differential equations (DDEs) with finitely many constant delays is presented. Using a generalization of the Lindstedt-Poincaré method to approximate a homoclinic solution allows us to initialize the continuation of the homoclinic bifurcation curves emanating from these points in DDEs. The normal form coefficients are derived in the functional analytic perturbation framework for dual semigroups (sun-star calculus) using a normalization technique based on the Fredholm alternative. The obtained expressions give explicit formulas, which have been implemented in the freely available numerical software package DDE-BifTool. The effectiveness is demonstrated on two models.

Stability and bifurcation for a class of retarded dynamical systems

Szilvia Császár and Sándor Kovács
Eötvös Loránd University, Hungary

In this work we study the qualitative behaviour of a linearized system of differential equation with positive coefficients and delay. The characteristic function of the system in question has the form

$$\Delta(z,\tau) :\equiv p(z) + q(z)e^{-z\tau} + r(z)e^{-2z\tau}$$

where p, q and r are polynomials with real coefficients and $\deg(r) > \deg(q)$. Using the Mikhailov criterion we give for special p, q and r fulfilling the above condition an estimates on the length of delay τ for which no stability switching occurs. Then for special parameters we compare our results with other methods. It follows a delay independent stability analysis. Finally, a formula for Hopf bifurcation is calculated in terms of p, q and r.

Global stability in difference equations

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We consider 2- and 3-dimensional maps depending on a parameter. Local stability of a fixed point is known up to a critical parameter value where Neimark-Sacker bifurcation takes place. The aim is to show global stability for all parameter values where local stability holds. Near the fixed point analytical tools are used to construct a neighbourhood $\mathcal{N}$ belonging to the domain of attraction of the fixed point. The size of the neighbourhood $\mathcal{N}$ is crucial since outside $\mathcal{N}$ rigorous, computer-aided calculations are applied to show that each point enters into $\mathcal{N}$ after finite number of iterations. The 3-dimensional case is technically more complicated as it requires a center manifold reduction, and in particular, an estimation on the size of center manifold is important. Among others, the difference equations

$$x_{k+1} = ax_k(1 - x_{k-d})$$

and

$$x_{k+1} = x_k e^{a - x_{k-d}}$$

where a is a positive parameter and $d = 1$ or $d = 2$, can be handled with our technique.

Oscillation constants for Euler type differential equations with $p(t)$-Laplacian

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It is well known that the oscillation problem for Euler type differential equation involving the p-Laplacian has a critical value. The critical value divides oscillation and nonoscillation of solutions of this equation, and it is called an oscillation constant. In this talk, we consider the oscillation problem for nonlinear ordinary differential equations involving the $p(t)$-Laplacian. The $p(t)$-Laplacian is a non-standard growth operator, and it represents a non-homogeneity and possesses more nonlinearity. Although the $p(t)$-Laplacian causes complexity, sufficient conditions are established for this equation to be (non)oscillatory. Obtained results, which are oscillation and nonoscillation theorems, shows that the oscillation problem for this equation has an oscillation constant. Moreover, we propose an open problem concerning the oscillation constant in the case when $p(t)$ tends to infinity as $t \to \infty$.

On the solutions of a second-order difference equation in terms of generalized Padovan sequences

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Julius Fergy T. Rabago
College of Science, University of the Philippines, PHILIPPINES

This paper deals with the solution, stability character and asymptotic behavior of the rational difference equation

$$x_{n+1} = \frac{\alpha x_{n-1} + \beta}{\gamma x_n x_{n-1}}, \qquad n \in \mathbb{N}_0,$$

where $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$, $\alpha, \beta, \gamma \in \mathbb{R}^+$, and the initial conditions x_{-1} and x_0 are non zero real numbers such that their solutions are associated to generalized Padovan numbers. Also, we investigate the two-dimensional case of the this equation given by

$$x_{n+1} = \frac{\alpha x_{n-1} + \beta}{\gamma y_n x_{n-1}}, \qquad y_{n+1} = \frac{\alpha y_{n-1} + \beta}{\gamma x_n y_{n-1}}, \qquad n \in \mathbb{N}_0.$$

Sharp estimation for the solutions of inhomogeneous delay differential and Halanay-type inequalities

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We consider inhomogeneous Halanay-type inequalities together with inhomogeneous linear delay differential inequalities and equations. Based on the the variation of constants formula and some results borrowed from a recent paper, sharp conditions for the boundedness and the existence of the limit of the nonnegative solutions are established. The sharpness of the results are illustrated by examples and by comparison of results in some earlier works.

Numerical methods for chaotic fluid mixing

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An approach to subgrid scale modeling is presented for chaotic problems involving a high degree of mixing over rapid time scales. The Richtmyer-Meshkov unstable flows are typical of such problems and are studied through the front tracking method in spherical geometry. The growth rate of fingers at an unstable shell driven by an imploding spherical shock is studied through simulations. An analysis is presented to support the assertion that the lower acceleration rate found in untracked simulations is caused by a reduced buoyancy force due to numerical mass diffusion across the interface.

Sign-constancy of Green's functions for two-point boundary value problems with second order impulsive differential equations

Alexander Domoshnitsky and Iuliia Mizgireva
Ariel University, Israel

We consider the following second order impulsive differential equation with delays

$$\begin{cases} (Lx)(t) \equiv x''(t) + \sum_{j=1}^{p} a_j(t)x'(t-\tau_j(t)) + \sum_{j=1}^{p} b_j(t)x(t-\theta_j(t)) = f(t), \\ \qquad t \in [0,\omega], \\ x(t_k) = \gamma_k x(t_k - 0), \quad x'(t_k) = \delta_k x'(t_k - 0), \quad k = 1, 2, ..., r. \end{cases}$$

We obtain sufficient conditions of nonpositivity of Green's functions for impulsive differential equation without an assumption about the sign of $b_j(t)$.

All results are formulated in the form of theorems about differential inequalities. Then choosing the test functions, we obtain explicit conditions of nonpositivity of Green's functions.

Fractional calculus in the context of distributional Henstock-Kurzweil integral

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The fractional derivatives is a branch of Mathematics which has implications in many areas as physics, chemistry, engineering, finance, among others. The fractional calculus and in particular, the Riemann-Liouville derivative have been developed in the context of Lebesgue integral. This integral is characterized in terms of absolutely continuous functions, AC, it means, F is an absolutely continuous function if and only if there exists f Lebesgue integrable function such that $F(x) = \int_a^x f + F(a)$, then $F'(x) = f(x)$ a.e. However, if F is a continuous function, then the generalized function and the distributional derivative are needed, because of is well known that there are a lot of continuous functions that are differentiable nowhere. The distributional Henstock-Kurzweil integral is defined by generalized functions, and this integral contains properly the Riemann, Lebesgue, Henstock-Kurzweil, Perron, Denjoy, and improper integrals.

In this work we define a generalized version of the Riemann-Liouville integral and derivative (in terms of the distributional Henstock-Kurzweil integral) and get new properties, for example, a general Fundamental Theorem of Calculus, semigroup property of the Riemann-Liouville integral operators, basic properties of the fractional derivative, and relations between the Riemann-Liouville derivative and integral.

Application of delay and advanced difference equations in insurance mathematics

Éva Orbán-Mihálykó, Csaba Mihálykó and András Győrfi-Bátori
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Sparre Andersen Risk Models are frequently investigated in insurance mathematics for modelling the operation of insurance companies. The most important questions are the probability and expected time of ruin in the function of the initial capital, and also the expectation of the deficit of the ruin. In the literature, usually accepted tool for this purpose is the so called Gerber-Shiu discounted penalty function.

The practical problems are usually discrete and the results for the discrete Sparre Andersen risk models are simpler to understand than their continuous analogues. Nevertheless, the discrete models have not attracted much attention and the literature counts fewer contributions. The continuous model lead to integral equations, the discrete ones result in difference equations.

In this presentation we investigate a discrete Sparre Andersen model allowing dependent inter-claim times and claim amounts. Moreover, we allow general premium rate. We define an auxiliary function which corresponds to the Gerber-Shiu discounted penalty function. We set up difference equations containing delay and advances too and we analyse these equations. We investigate the existence and uniqueness of the solutions. In some special cases we provide explicit solutions. In this presentation we also deal with the moments of the deficit of ruin.

Hille-Nehari type criteria for conditionally oscillatory half-linear differential equations

Zuzana Pátíková
Tomas Bata University in Zlín, Czech Republic

Considering a nonoscillatory half-linear second order differential equation

$$(r(t)\Phi(x'))' + c(t)\Phi(x) = 0, \quad \Phi(x) = |x|^{p-1} \operatorname{sgn} x, \quad p > 1,$$

with the use of its solution h, it is possible to construct its perturbation so that the resulting equation is conditionally oscillatory. We present Hille-Nehari type integral oscillation and nonoscillation criteria for the perturbed conditionally oscillatory equation with the critical coefficient

$$(r(t)\Phi(x'))' + \left[c(t) + \frac{1}{2qh^p(t)R(t)(\int^t R^{-1}(s)\ \mathrm{d}s)^2} + \tilde{c}(t) \right] \Phi(x) = 0$$

and we show, what is the form of the next perturbation which makes the perturbed equation again conditionally oscillatory.

Distributed feedback control in model of testosterone regulation

Olga Pinhasov and Alexander Domoshnitsky
Ariel University, Israel

We propose a model describing stabilization of testosterone by distributed input feedback control. We aimed to hold testosterone concentration above the corresponding level. The feedback control with integral term is proposed. This leads us to the stability analysis of the system

$$\begin{cases} x_1'(t) + b_1 x_1(t) = 0, \\ x_2'(t) + b_2 x_2(t) - g_1 x_1(t) + c_2 \displaystyle\int_0^t e^{-\alpha_2(t-s)} x_3(s) \mathrm{d}s = 0, \\ x_3'(t) + b_3 x_3(t) - c_1 \displaystyle\int_0^t e^{-\alpha_1(t-s)} x_2(s) ds + c_3 \displaystyle\int_0^t e^{-\alpha_3(t-s)} x_3(s) ds = 0. \end{cases}$$

The role of the time delay in the reflection and transmission of ultrashort electromagnetic pulses on a system of parallel current sheets

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In this lecture we discuss how we derived from first principles a coupled system of equations describing the scattering of plane electromagnetic radiation fields on two parallel metal layers, which are embedded in three dielectric media. In this description the radiation field may represent e.g. ultrashort light pulses of arbitrary temporal shape and intensity, within the limit of the non-relativistic description of the local electron motions. This formalism yields a closed coupled set of delay differential-difference equations for the reflected and transmitted field components and for the electronic velocities in the layers. Our approach to solve the resulting system is to consider the singularly perturbed system

$$\frac{d}{dt}(E(\epsilon)x(t)) = Ax(t) + Bx(t-\tau) + Cx(t-2\tau) + h\,(t)\,, \quad t \geq 0,$$

and find necessary and sufficient conditions for which the solution of the perturbed system approaches the solution of the unperturbed system as $\epsilon \to 0^+$.

The Sturm separation theorem for impulsive delay differential equations

Alexander Domoshnitsky and Vladimir Raichik
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Wronskian is one of the classical objects in the theory of ordinary differential equations. Properties of Wronskian lead to important conclusions on behavior of solutions of delay equations.
For instance, non-vanishing Wronskian ensures validity of Sturms separation theorem(between two adjacent zeros of a solution there is one and only one zero of every other nontrivial linearly independent solution) for delay equations.
We propose the Sturm separation theorem in case of impulsive delay differential equations and obtain assertions about its validity for impulsive delay differential equations.

Numerical solution of fractional control problems via fractional differential transformation

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The subject of the study is a linear fractional control problem with constant delays in the state. Single-order systems with fractional derivative in Caputo sense of order between 0 and 1 are considered. The aim of the presentation is to introduce a new algorithm convenient for numerical approximation of a solution to the studied problem. The method consists of the fractional differential transformation in combination with the methods of steps. The original system is transformed to a system of recurrence relations. Approximation of the solution is given in the form of truncated fractional power series. The choice of order of the fractional power series is discussed and the order is determined in relation to the order of the system. An application on a two-dimensional fractional system is included to demonstrate efficiency and reliability of the algorithm.

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Linear differential equations of neutral type and retract method

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When studying the asymptotic properties of solution of functionally differential equations with delay, many authors use the retract principle. For a suitably chosen area, it is possible from the knowledge of the derivatives of the solution of the given equation to deduce the existence of at least one solution that remains in the given area for the argument going to infinity.Since the derivative of a concrete solution of a neutral type equation depends on its derivative at a delayed argument, a polyfatial set with respect to subsidiary inequality is also defined. This approach can be applied to linear equations. The possibility of using this method is shown to derive a new criterion of the existence of positive decreasing solutions of the linear equation

$$\dot{y}(t) = -c(t)y(t - \tau(t)) + d(t)\dot{y}(t - \delta(t))$$

where $c\colon [t_0, \infty) \to (0, \infty)$, $d\colon [t_0, \infty) \to (0, \infty)$, $t_0 \in \mathbb{R}$ and $\tau, \delta\colon [t_0, \infty) \to (0, r]$, $r \in \mathbb{R}$, $r > 0$ are continuous functions.

Stabilization by delay distributed feedback control

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Although stabilizing distributed feedback control systems is a challenging problem, only a few papers were devoted to it. Note in this connection the recent papers [2,3,5]. These works in a corresponding sence present non-trivial developments of the ideas of a reduction formulated in [1] (see also the finite spectrum assigment method [4]). A noise in the feedback delay control is one of the main reasons for developing mathematical models with distributed inputs: it is impossible to base our control on the value of the process $X(t_j)$ at a moment t_j only, we have to use an average value of the process $X(t)$ at a corresponding neighborhood of t_j. The integral term with a kernel defining a weight of every value takes on this role. It points out in [2] that such models with distributed inputs can appear in population dynamics, in propellant rocket motors and in networked control systems. Assertions on stability of integro-differential equations are proposed. On the basis of these results, new possibilities of distributed input control are proposed. Stabilization of this sort, according to common belief requires a damping term in the second order differential equation. Results obtained in this paper refute this delusion.

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Spectral theory of discrete symplectic systems

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In this talk we present our recent results concerning the initial development of the spectral theory of the discrete symplectic systems (in the time-reversed form)

$$z_k(\lambda) = \mathbb{S}_k(\lambda)\, z_{k+1}(\lambda), \qquad \mathbb{S}_k(\lambda) := \mathcal{S}_k + \lambda \mathcal{V}_k, \tag{S$_\lambda$}$$

where $\lambda \in \mathbb{C}$ is the spectral parameter and the $2n \times 2n$ complex-valued matrices $\mathcal{S}_k$ and $\mathcal{V}_k$ are such that

$$\mathcal{S}_k^* \mathcal{J} \mathcal{S}_k = \mathcal{J},\ \mathcal{V}_k^* \mathcal{J} \mathcal{S}_k \ \text{ is Hermitian},\ \mathcal{V}_k^* \mathcal{J} \mathcal{V}_k = 0,\ \Psi_k := \mathcal{J} \mathcal{S}_k \mathcal{J} \mathcal{V}_k^* \mathcal{J} \geq 0$$

with the skew-symmetric $2n \times 2n$ matrix $\mathcal{J} := \left(\begin{smallmatrix} 0 & I \\ -I & 0 \end{smallmatrix}\right)$, the superscript $*$ denoting the conjugate transpose, and k belonging to a discrete interval $\mathcal{I}$, which is finite or unbounded above. We discuss the (non-)existence of a densely defined operator associated with system (S$_\lambda$) and the problem of a characterization of self-adjoint extensions, which is closely connected with the number of square summable solutions of (S$_\lambda$).

Nonlinear analysis of a scalar implicit neutral delay differential equation

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Gabor Stepan
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We conduct Hopf bifurcation analysis on a scalar implicit Neutral Delay Differential Equation (NDDE) with two analytical methods: 1) normal form theory; 2) method of multiple scales. The modifications of the classical algorithms which are originally developed for explicit differential equations yields the same algebraic results. These results are further confirmed by numerical simulations. We show that the generalizations of these regular normal form calculation methods are useful for the local nonlinear analysis of implicit NDDEs where the explicit formalism is typically not accessible and the existence and uniqueness of solutions around the equilibrium are only assumed together with the existence of a smooth local center manifold.